TEST INFORMATION

DATE : 03.05.2015 **OPEN TEST (OT-02) ADVANCED**

Syllabus : Full Syllabus

REVISION DPP OF

DEFINITE INTEGRATION & ITS APPLICATION AND INDEFINITE INTEGRATION

Total Marks : 149 **Max. Time : 105.5 min.**

Single choice Objective (–1 negative marking) Q. 1 to 14	**(3 marks 2.5 min.)**	**[42, 35]**
Multiple choice objective (–1 negative marking) Q. 15 to 31	**(4 marks, 3 min.)**	**[64, 48]**
Comprehension (–1 negative marking) Q.32 to 33 & Q.34 to Q.36	**(3 marks 2.5 min.)**	**[15, 12.5]**
Single digit type (no negative marking) Q. 37 to 39	**(4 marks 2.5 min.)**	**[12, 7.5]**
Double digit type (no negative marking) Q. 40	**(4 marks 2.5 min.)**	**[16, 2.5]**

1. If A = $\int_{0}^{505\pi} |\cos x|\,dx$ and B = $\int_{505\pi}^{1007\pi} |\sin x|\,dx$, then A + B is equal to

(A) 2013 (B) 2014

(C) 2015 (D) 2016

2. The least integer greater than $\int_{0}^{100} \left\{\sqrt{x}\right\} dx$ is (where {.} is fractional part function)

(A) 50 (B) 51

(C) 52 (D) 53

3. $\int \frac{e^x\left(2-x^2\right)}{(1-x)\sqrt{1-x^2}}\,dx$ =

(A) $e^x \cdot \sqrt{\frac{1-x}{1+x}}$ + c (B) $e^x\sqrt{\frac{1+x}{1-x}}$ + c

(C) $\frac{e^x}{\sqrt{1-x}}$ + c (D) $\frac{e^x}{\sqrt{1+x}}$ + c

4. Let f(x) = $\int x^2\cos^2 x\left(2x+6\tan x-2x\tan^2 x\right)dx$ and f(x) passes through the point (π, 0), then the number of solutions of the equation f(x) = x^3 in [0, 2π] is

(A) 1 (B) 2 (C) 3 (D) 4

5. Let f(x) is a continuous function symmetric about the lines x = 1 and x = 2. If $\int_0^2 f(x)dx = 3$ and $\int_0^{50} f(x)\ dx = \mathrm{I}$, then $\left[\sqrt{\mathrm{I}}\right]$ is equal to (where [.] is G.I.F.)

(A) 5 (B) 8 (C) 7 (D) 6

6. $\int\left(x^6+x^4+x^2\right)\sqrt{2x^4+3x^2+6}\ \ dx$ is equal to

(A) $\frac{\left(3x^6+2x^4+6x^2\right)^{3/2}}{18}+C$

(B) $\frac{\left(2x^6+3x^4+6x^2\right)^{3/2}}{24}+C$

(C) $\frac{\left(2x^6+3x^4+6x^2\right)^{3/2}}{18}+C$

(D) None of these

7. For each positive integer n > 1, let S_n represents the area of the region bounded by $\frac{x^2}{n^2}+y^2\le 1$ and $x^2+\frac{y^2}{n^2}\le 1$, then $\lim\limits_{n\to\infty} S_n$ is equal to

(A) 4 (B) 1 (C) 2 (D) 3

8. $\int\frac{8x^{43}+13x^{38}}{\left(x^{13}+x^5+1\right)^4}dx$ =

(A) $\frac{x^{39}}{3\left(x^{13}+x^5+1\right)^3}$ + c

(B) $\frac{x^{39}}{\left(x^{13}+x^5+1\right)^3}$ + c

(C) $\frac{x^{39}}{5\left(x^{13}+x^5+1\right)^5}$ + c

(D) $\frac{x^{52}}{3\left(x^{13}+x^5+1\right)}$ + c

9. Let $\int e^x(f(x)-f'(x))dx = \phi(x)$, then $\int e^x f(x)dx$ is equal to

(A) $\phi(x) + e^x f(x) + c$

(B) $\phi(x) - e^x f(x) + c$

(C) $\frac{1}{2}\{\phi(x) + e^x f(x)\} + c$

(D) $\frac{1}{2}(\phi(x) + e^x f'(x)) + c$

10. Suppose f(x) is a real valued differentiable function defined on $[1, \infty)$ with f(1) = 1. Further let f(x) satisfy $f'(x) = \frac{1}{x^2 + f^2(x)}$, then the range of values of f(x) is

(A) $[1, \infty)$ (B) $[1, 1 + \pi/4)$

(C) $[1, \pi/4)$ (D) $[1 - \pi/4, 1]$

11. The value of $\lim_{x \to 0} \frac{\int_0^x x e^{t^2} dt}{1 + x - e^x}$ is equal to

(A) 1 (B) 2 (C) –1 (D) –2

12. Let f(x) be a differentiable function such that f(0) = 0 and $\int_0^2 f'(2t)\, e^{f(2t)} dt = 5$, then the value of f (4) equals

(A) 2 ℓn3 (B) ℓn10 (C) ℓn11 (D) 3 ℓn2

13. The area enclosed by the curve $y \le \sqrt{4 - x^2}$, $y \ge \sqrt{2} \sin\left(\frac{\pi x}{2\sqrt{2}}\right)$ and the x-axis is divided by y-axis in the ratio

(A) $\frac{\pi^2 - 8}{\pi^2 + 8}$ (B) $\frac{\pi^2 - 4}{\pi^2 + 4}$ (C) $\frac{\pi - 3}{\pi + 4}$ (D) $\frac{2\pi^2}{2\pi + \pi^2 - 8}$

14. For any $t \in R$ and f being a continuous function.

Let $I_1 = \int_{\sin^2 t}^{1 + \cos^2 t} x f\left(x(2 - x)\right) dx$

$I_2 = \int_{\sin^2 t}^{1 + \cos^2 t} f\left(x(2 - x)\right) dx$, then

(A) $I_1 = I_2$ (B) $I_1 = 2I_2$ (C) $2I_1 = I_2$ (D) $I_1 + I_2 = 0$

15. If $\int_2^x g(t)\,dt = \frac{x^2}{2} + \int_x^2 t^2 g(t)\,dt$, then equation $g(x) = \lambda$ has

(A) 2 solution if $|\lambda| < \frac{1}{2}$ (B) 2 solution if $|\lambda| < \frac{1}{2}$ & $\lambda \ne 0$

(C) 1 solution if $\lambda = -\frac{1}{2}$ (D) No solution if $|\lambda| > \frac{1}{2}$

16. If g(x) = $\{x\}^{[x]}$, where {.} and [.] represents fractional part and greatest integer function respectively and

$f(k) = \int_{k}^{k+1} g(x)dx$ $(k \in N)$, then

(A) f(1), f(2), f(3), ... are in H.P.

(B) $\sum_{r=1}^{\infty}(-1)^{r+1}$ f(r) = $1 - \ell n\, 2$

(C) $\sum_{r=1}^{\infty}(-1)^{r}$ f(r) = $\ell n\left(\frac{2}{e}\right)$

(D) $\sum_{r=0}^{n} f\left(\frac{1}{r}\right) = \frac{n(n+1)}{2}$

17. If f(x) is a differentiable function such that f(x + y) = f(x) f(y) $\forall$ x, y $\in$ R, f(0) $\neq$ 0 and

$g(x) = \frac{f(x)}{1+(f(x))^2}$, then

(A) $\int_{-2014}^{2015} g(x)dx = \int_{0}^{2015} g(x)dx$

(B) $\int_{-2014}^{2015} g(x)dx - \int_{0}^{2014} g(x)dx = \int_{0}^{2015} g(x)dx$

(C) $\int_{-2014}^{2015} g(x)dx = 0$

(D) $\int_{-2014}^{2014} 2g(-x) - g(x)\, dx = 2\int_{0}^{2014} g(x)dx$

18. Let $I = \int_{-1}^{2}\left(\cot^{-1}\frac{1}{x} + \cot^{-1}x\right)dx$ and $J = \int_{-2\pi}^{7\pi}\frac{\sin x}{|\sin x|}dx$, then which of the following is/are correct ?

(A) 2I + J = 6π

(B) 2I – J = 3π

(C) $4I^2 + J^2 = 26\pi^2$

(D) $\frac{I}{J} = \frac{5}{2}$

19. If $I_n = \int_{0}^{\pi}\frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\frac{x}{2}}dx$, where n $\in$ w, then

(A) $I_{n+2} = I_n$

(B) $\sum_{m=1}^{10} I_m = 10\pi$

(C) $\sum_{m=1}^{10} I_{2m-1} = 10\pi$

(D) $I_{n+1} = I_n$

20. If $\int_0^1 \frac{x^4\left(1+x^{10065}\right)}{\left(1+x^5\right)^{2015}}\,dx = \frac{1}{p}$, then

(A) Number of ways in which p can be expressed as a product of two relatively prime factors is 8.

(B) Number of ways in which p can be expressed as a product of two relatively prime factors is 4.

(C) Number of ways in which p can be expressed as a product of two factors is 8.

(D) Number of ways in which p can be expressed as a product of two factors is 4.

21. If $T_n = \sum_{r=2n}^{3n-1} \frac{r}{r^2+n^2}$ and $S_n = \sum_{r=2n+1}^{3n} \frac{r}{r^2+n^2}$ $\forall\, n \in \{1, 2, 3, \ldots\}$, then

(A) $T_n > \ell n\sqrt{2}$

(B) $S_n < \ell n\sqrt{2}$

(C) $T_n < \ell n\sqrt{2}$

(D) $S_n > \ell n\sqrt{2}$

22. If $I_1 = \int_0^1 \frac{1+x^8}{1+x^4}dx$ and $I_2 = \int_0^1 \frac{1+x^9}{1+x^3}dx$, then

(A) $I_2 < I_1 < \pi/4$

(B) $\pi/4 < I_2 < I_1$

(C) $1 < I_1 < I_2$

(D) $I_2 < I_1 < 1$

23. Consider a continuous function 'f' where $x^4 - 4x^2 \le f(x) \le 2x^2 - x^3$ such that the area bounded by $y = f(x)$, $g(x) = x^4 - 4x^2$, the y-axis and the line $x = t$ $(0 \le t \le 2)$ is twice of the area bounded by $y = f(x)$, $y = 2x^2 - x^3$, y-axis and the line $x = t$ $(0 \le t \le 2)$ then

(A) $f(2) = 0$

(B) $f(1) = 1/3$

(C) $f'(1) = -2/3$

(D) f(x) has two points of extrema

24. The value of the definite integral $\int_2^4 \left(x(3-x)(4+x)(6-x)(10-x) + \sin x \right)dx$ equals

(A) cos 2 + cos 4

(B) cos 2 – cos 4

(C) 2cos1 cos3

(D) 2sin1 sin 3

25. The value of the definite integral $\int_{-\infty}^{a} \frac{(\sin^{-1}e^x + \sec^{-1}e^{-x})dx}{(\cot^{-1}e^a + \tan^{-1}e^x)(e^x + e^{-x})}$ $(a \in R)$ is

(A) Independent of a

(B) dependent on a

(C) $\frac{\pi}{2}\ell n2$

(D) $-\frac{\pi}{2}\ell n\left(\frac{2}{\pi}\tan^{-1}e^{-a}\right)$

26. Let $I = \int_{k\pi}^{(k+1)\pi} \frac{|\sin 2x|dx}{|\sin x| + |\cos x|}$, $(k \in N)$ and $J = \int_0^{\pi/4} \frac{dx}{\sin x + \cos x}$, then which of the following hold(s) good ?

(A) $I = 2\int_0^{\pi/2} \frac{\sin 2x\, dx}{\sin x + \cos x}$

(B) $I = 4 - 4J$

(C) $I = 4 - 2J$

(D) $I = 2 - 2J$

27. If $f(x) = \int \frac{x^8 + 4}{x^4 - 2x^2 + 2}\, dx$ and $f(0) = 0$, then

(A) f(x) is an odd function

(B) f(x) has range R

(C) f(x) = 0 has at least one real root

(D) f(x) is a monotonic function

28. If $f(x) = \int_0^{\pi/2} \frac{\ell n\left(1 + x\sin^2\theta\right)}{\sin^2\theta} d\theta$, $x \geq 0$, then

(A) $f(x) = \pi\left(\sqrt{x+1} - 1\right)$

(B) $f'(3) = \frac{\pi}{4}$

(C) f(x) cannot be determined

(D) $f'(0) = \frac{\pi}{2}$

29. If $f : R \to R$ be a continuous function such that $f(x) = \int_1^x 2tf(t)dt$, then which of the following does not hold(s) good?

(A) $f(\pi) = e^{\pi^2}$

(B) $f(1) = e$

(C) $f(0) = 1$

(D) $f(2) = 2$

30. If $\lim_{n\to\infty} \sum_{r=1}^{n} \left(\left(\frac{3r}{n}\right)^2 + 2\right)\frac{3}{n} = \int_0^b f(x)dx$, then

(A) $b = 1$

(B) $f(x) = 9x^2 + 6$

(C) $\lim_{n\to\infty} \sum_{r=1}^{n} \left(\left(\frac{3r}{n}\right)^2 + 2\right)\frac{3}{n} = 9$

(D) $\lim_{n\to\infty} \sum_{r=1}^{n} \left(\left(\frac{3r}{n}\right)^2 + 2\right)\frac{3}{n} = 15$

31. A real valued function $f(x) : R^+ \to R^+$ satisfies $\int_0^1 f(tx)dt = nf(x)$. If $\lim_{n\to\infty} f(x) = g(x)$, $g(1) = 2$ and area bounded by $y = g(x)$ with x-axis from $x = 3$ to $x = 7$ is S, then

(A) $S \in \left(2, \frac{8}{3}\right)$

(B) $S \in \left(\frac{8}{7}, \frac{8}{3}\right)$

(C) $S < \frac{40}{21}$

(D) $S > \ell n4$

Comprehension # 1 (For Q. No. 32 to 33)

Consider the integral $I = \int_{0}^{10\pi} \frac{\cos 4x \cos 5x \cos 6x \cos 7x}{1+e^{2\sin 2x}} dx$

32. If $I = k . \int_{0}^{\pi/2} \cos 4x \cos 5x \cos 6x \cos 7x \, dx$, then 'k' is equal to

(A) 5 (B) 10 (C) 1 (D) 20

33. If $I = \lambda . \int_{0}^{\pi/4} \cos 2x \cos 4x \cos 6x \, dx$, then 'λ' is equal to

(A) 5 (B) 20 (C) 10 (D) 5/2

Comprehension # 2 (For Q. No. 34 to 36)

For i = 0, 1, 2, ... , n, let S_i denotes the area of region bounded by the curve $y = e^{-2x} \sin x$ with x-axis from $x = i\pi$ to $x = (i + 1)\pi$.

34. The value of S_0 is

(A) $\frac{1+e^{2\pi}}{5}$ (B) $\frac{1-e^{-2\pi}}{5}$

(C) $\frac{1+e^{-2\pi}}{5}$ (D) $\frac{1+e^{-\pi}}{5}$

35. The ratio $\frac{S_{2014}}{S_{2015}}$ is equal to

(A) $e^{-2\pi}$ (B) $e^{2\pi}$ (C) $2e^{\pi}$ (D) $e^{-\pi}$

36. The value of $\sum_{i=0}^{\infty} S_i$ is equal to

(A) $\frac{e^{\pi}\left(1+e^{\pi}\right)}{5\left(e^{\pi}-1\right)}$ (B) $\frac{e^{2\pi}\left(e^{2\pi}+1\right)}{5\left(e^{2\pi}-1\right)}$

(C) $\frac{e^{2\pi}+1}{5\left(e^{2\pi}-1\right)}$ (D) $\frac{e^{2\pi}+1}{e^{2\pi}-1}$

37. Let f(x) be differentiable function satisfying the condition $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$ $\forall$ x, y $\in$ R – {0} and f(x) $\neq$ 0, f′(1) = 2. If the area enclosed by y $\geq$ f(x) and $x^2 + y^2 \leq 2$ is A, then find [2A], where [.] represents G.I.F.

38. The value of the definite integral $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{(\sin x+\cos x+2\sqrt{\sin x\cos x})\sqrt{\sin x\cos x}}$ equals

39. A continuous real function 'f' satisfies f(2x) = 3 f(x) $\forall$ x $\in$ R. If $\int_0^1 f(x)dx = 1$, then compute the value of definite integral $\int_1^2 f(x)dx$

40. If $2^{2010}\frac{\int_0^1 x^{1004}(1-x)^{1004}\,dx}{\int_0^1 x^{1004}\left(1-x^{2010}\right)^{1004}\,dx}$ = λ, then find the highest prime factor of λ.

ANSWER KEY
DPP # 7

REVISION DPP OF
VECTORS AND THREE DIMENSIONAL GEOMETRY

1.	(C)	**2.**	(C)	**3.**	(B)	**4.**	(B)	**5.**	(A)	**6.**	(A)	**7.**	(A)
8.	(C)	**9.**	(A)	**10.**	(C)	**11.**	(B)	**12.**	(C)	**13.**	(C)	**14.**	(B)
15.	(C)	**16.**	(B)	**17.**	(A)	**18.**	(A,D)	**19.**	(B,D)	**20.**	(B,D)		
21.	(B,C,D)	**22.**	(B,C)	**23.**	(B,C)	**24.**	(A,B,C)	**25.**	(A,B,C,D)	**26.**	(A,B,D)		
27.	(A,C,D)	**28.**	(A,C,D)	**29.**	(A,B)	**30.**	(A,C,D)	**31.**	(A,C,D)	**32.**	(A,B,D)		
33.	(A,C,D)	**34.**	(C,D)	**35.**	(B,D)	**36.**	(A,B,D)	**37.**	(A,D)	**38.**	(D)	**39.**	(C)
40.	(B)												